Global and Regional Gravity Field Solutions from GRACE Observations

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Abstract. A procedure for gravity field determination is presented based on the analysis of short arcs of a twin satellite configuration as realized in the GRACE twin satellite mission. The mathematical setup is based on the solution of the equation of motion of the two satellites as a boundary value problem and formulated as integral equation. The method can be applied to a global gravity field recovery and this global model can be refined in regions with rough gravity field features or in specific areas of scientific interest. The global gravity field is modeled by a series of spherical harmonics, as usual, and the regional gravity field refinements are represented by localizing base functions defined on a spherical grid based on icosahedral triangles. A simulation scenario has been generated to investigate the proposed approach. First results with real data for a global recovery and regional refinements are presented.


1 Introduction

The CHAMP and GRACE space-borne gravity field missions, which have been in orbit since 2000 and 2002 respectively, improve our knowledge of the Earth’s gravity field in the long- and medium-wavelength ranges. The innovative character of these missions lies in the continuous observation by the Global Positioning System (GPS) and the highly precise line-of-sight range-rate measurements by GRACE. Several new approaches are presented for gravity field recovery by analysis of precisely determined orbits (POD). Our approach is to divide the orbit into short arcs and formulate the functional model as integral equation for the solution of the equation of motion. This approach has been successfully tested for the satellite mission CHAMP, see Mayer-Guerr et al. (2004). So this method will now be expanded to range and range-rate measurements provided by the GRACE K-band microwave link.

2 Mathematical Setting

The mathematical model of the gravity field recovery technique is based on Newton’s equation of motion,

\[ \ddot{r}(t) = f(t, r, \dot{r}) \]  

where the vectors \( r, \dot{r} \) and \( \ddot{r} \) denote position, velocity and acceleration of the satellite, referred to a (quasi-)inertial reference system. The function \( f \) indicates the specific (mass-related) force function acting on the satellite. This force-function contains the unknown earth gravity but also tides from sun, moon and other planets, earth tides, ocean tides and non conservative surface forces. The latter ones are measured by the onboard accelerometers.

The primary observation of the GRACE mission are functionals of the relative motion of the two GRACE satellites, relative distances and range-rates. But also the precisely simultaneously determined orbits of both satellites based on GPS observations contribute to the gravity field recovery. The observation equation for the latter one is the solution of Eq. (1), formulated as a boundary value problem, according to Schneider (1968),

\[ r(\tau) = (1 - \tau)r_A + \tau r_B + T^2 \int_0^1 K(\tau, \tau')f(\tau', r, \dot{r}) \, d\tau', \]  

with the normalized time,

\[ \tau = (t - t_A)/T, \quad T = t_B - t_A, \]
and the integral kernel
\[ K(\tau, \tau') = \begin{cases} \tau'(1 - \tau) & \text{for } 0 \leq \tau' \leq \tau \\ \tau(1 - \tau') & \text{for } \tau \leq \tau' \leq 1 \end{cases} \]

The equation is applied to short arcs of about 30 min in which the boundary values \( r_A \) and \( r_B \) are treated as unknowns. The gravity field in the force function \( f \) is splitted in an known reference part and an unknown disturbance part. The disturbing field is modelled either by a spherical harmonic expansion with coefficients \( \Delta c_{nm} \) and \( \Delta s_{nm} \) or in case of regional refinements by space localizing base functions of harmonic spline type, see Freeden et al. (1998).

To get the functional model of the primary GRACE observations range \( \rho_{12} \) and range-rate \( \dot{\rho}_{12} \), Eq. (2) is formulated as relative motion between the two satellites:
\[ r_{12}(\tau) = (1 - \tau)r_{12A} + \tau r_{12B} + T^2 \int_0^1 K(\tau, \tau') f_{12}(\tau', r, \dot{r}) d\tau', \quad (4) \]

and for the relative velocity:
\[ \dot{r}_{12}(\tau) = \frac{1}{T} (r_{12B} - r_{12A}) + T \int_0^1 \frac{\partial K(\tau, \tau')}{\partial \tau} f_{12}(\tau', r, \dot{r}) d\tau'. \quad (5) \]

The range observations themselves can be written as
\[ \rho_{12}(\tau) = e_{12}(\tau) \cdot r_{12}(\tau), \quad (6) \]

and range-rate measurements as
\[ \dot{\rho}_{12}(\tau) = e_{12}(\tau) \cdot \dot{r}_{12}(\tau), \quad (7) \]

where \( e_{12} \) is a unity vector in the line-of-sight direction. This vector is known with high accuracy assuming that the satellite positions are measured with an accuracy of a few cm and taking into account the distance of over 200 km between the two satellites. For example a positon error of 3 cm of both satellites results in a line-of-sight direction error of 0.04. Therefore the functional models for range observations or range-rate observations are Eq. (4) or Eq. (5) projected into the line-of-sight direction.

3 Normal equations and Observation Weighting

For analysing the GRACE data not only the gravity field parameters have to be estimated, but also arc-related parameters have to be determined. There are at least six parameters per arc which are the boundary position vectors. Assuming an arc length of 30 min they are accumulated to over 9000 unknowns per month. To reduce the size of the normal equations, these arc-related parameters are eliminated before the arcs are merged to the complete system of normal equations. In the observation equation for each arc
\[ l = (A_g, A_a)(x_g, x_a)^T + e, \quad (8) \]

the unknowns are split in gravity field parameters \( x_g \) and arc-related parameters \( x_a \). I describes the the measurements vector and \( e \) the residui vector. From the normal equation system
\[ N = (A_g^T A_g A_a^T A_a)^{-1} A_g^T A_a, \]

the arc-related parameters \( x_a \) can easily be eliminated by
\[ N = A_g^T A_g - A_g^T A_a (A_a^T A_a)^{-1} A_a^T A_g, \quad (10) \]

and
\[ \bar{b} = A_g^T 1 - A_g^T A_a(A_a^T A_a)^{-1} A_a^T 1. \quad (11) \]

The reduced normal equations are computed per arc and accumulated. To make the solution robust against less accurate periods of the orbit, a variance component estimation (VCE) procedure is used. This can be done efficiently by re-weighting every orbital arc individually
\[ N = \sum \frac{1}{\sigma_i^2} \tilde{N}_i, \quad b = \sum \frac{1}{\sigma_i^2} \tilde{b}_i. \quad (12) \]

The variance factors can be computed in an iterative approach by
\[ \sigma_i^2 = \frac{e_i^T e_i}{r_i} \quad (13) \]

with the residuals
\[ e = 1 - A_g x_g - A_a x_a \quad (14) \]

and partial redundancy
\[ r_i = n_i - u_i - \frac{1}{\sigma_i} \text{trace}(\tilde{N}_i N^{-1}), \quad (15) \]

where \( n_i \) is the number of observations and \( u_i \) is the number of eliminated parameters. The trace can efficiently be computed by a Monte Carlo method, see Koch and Kusche (2001).

4 Simulation Results

To investigate the recovery procedure, a simulation scenario is generated. The nearly circular orbits of the two GRACE satellites are computed at an altitude of 490km, following each other in a distance of approximately 230km. The pseudo real gravity field is simulated by a spherical harmonic expansion complete up to degree 90. The range rate observations are generated every 5 seconds covering a 30 days mission period and are corrupted by white noise with an rms of 1 \( \mu \)m/s, and the satellite positions of both satellites are corrupted by white noise with an rms of 3 cm. Range observations are not used for this test. The orbits are split in 1500 short arcs of approximately 30 min duration. The unknown gravity field is represented by a spherical harmonic expansion from degree 2 to degree 90 leading to 8277 unknown parameters.
The differences between the recovery results and the pseudo–real gravity field up to degree \( n = 70 \) are shown in Fig. 1. It reveals a maximum error of 4.4 mm, an average deviation of 0.8 mm and an rms of 1.0 mm in terms of geoid heights.

To take a closer look at the results the degree amplitudes of the signal, the errors and the estimated errors are displayed in Fig. 2. From these graphs it becomes obvious that the possible resolution for gravity field recovery is not limited to degree \( n = 90 \). The limit of resolution would be reached at the point where line of error degree amplitude intersects with the line of signal degree amplitudes. At degree \( n = 90 \) the error is still magnitudes smaller than the signal amplitude. Fig. 2 shows a difference between the real and the estimated error. This discrepancy cannot be explained at the moment.

5 Real Data Results

5.1 Global Gravity Field Recovery

The same approach is applied to real SST range-rate measurements based on one month of observations from July 2003. Range observations are not used. The data are corrected for the tides from sun, moon and other planets. The ephemeris are taken from the JPL 405 data set. Effects originating from the deformation of the Earth caused by these tides are modelled following the IERS 2003 conventions. Ocean tides are computed from the FES2004 model. Effects of high frequency atmosphere and ocean mass redistributions are removed by the GFZ AOD dealiasing product. The orbits are split in 1500 short arcs of approximately 30 min duration. For each arc the boundary values and an accelerometer bias are estimated. The unknown gravity field is represented by a spherical harmonic expansion from degree 2 to degree 90 leading to 8277 unknown parameters.

In order to validate the result our solution is compared to the first published GRACE gravity field model GGM01S (Tapley et al. (2004)). The differences between the recovery results and the GGM01S gravity field in terms of geoid heights up to degree \( n = 70 \) are shown in Fig. 3. It reveals a maximum error of 20.0 cm, an average deviation of 2.6 cm and an rms of 3.4 cm. Furthermore a comparison on the level of the degree amplitudes is performed, as illustrated in Fig. 4. First of all it becomes evident that the differences to GGM01S are more than ten times larger than the errors in the simulation scenario, visible as well in the geoid heights as in the degree amplitudes. This large effect cannot only attributed to our solution as the differences (red line in
Fig. 4) are only slightly larger than the estimated errors of the GGM01S (black dotted line in Fig. 4). In our opinion the discrepancies could be the result of inaccurate ocean tide and dealiasing models. When evaluating the results it has to be taken into account that the GGM01S is based on the analysis of 111 days of GRACE data whereas we only used 30 days of data.

5.2 Regional Refinements

Based upon this solution a regional refinement is calculated for the Himalaya region and South-America. The same data set and the same observation equations are used to determine the regional solutions. The only difference is the parameterization of the gravity field, for the global solution a spherical harmonic expansion and for the regional solutions space localizing base functions. The regional refinements are determined with a resolution resembling a spherical harmonic expansion up to degree 120, calculated as residual fields to the global solution, for details concerning this refinement procedure see Eicker et al. (2005) and Ilk et al. (2005). For the regional gravity field recovery only the satellite data over the respective area is used. To avoid geographical truncation effects at the boundaries of the regions, an additional border of 10° is taken into account.

The results are presented in Fig. 5 and Fig. 6. For the Himalaya region it reveals a maximum error of 89.5 cm, an average deviation of 19.4 cm and an rms of 24.3 cm in terms of geoid heights. and for South-America a maximum error of 57.0 cm, an average deviation of 10.1 cm and an rms of 12.9 cm. The regional refinement procedure obviously does not achieve any significant improvement. The reason for this might be the fact that the regional solutions are influenced by the same ocean tide and dealiasing models. Therefore possible inaccuracies in these models affect the regional solutions as well.

6 Conclusions

The use of short arcs for gravity field recovery, based on the solution of Newtons equation of relative motion, formulated as a boundary value problem, is an adequate recovery technique tailored to the SST observables of the type range and range-rate. The simulation results indicate the practicability of this approach. First results are presented using real SST range-rate measurements based on the observations of July 2003. The results are not yet satisfactory. Therefore an improvement of the recovery procedure and of the used models is necessary. After solving these remaining difficulties we expect our approach to deliver a competitive global GRACE gravity field model. Furthermore we anticipate our regional refinement strategy to enable further improvement of the global model.

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