A new CHAMP gravitational field model based on the GIS acceleration approach and two years of kinematic CHAMP data

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Abstract. A new CHAMP gravity field model is presented based on the acceleration approach and two years of kinematic CHAMP orbit data. Basically, acceleration vectors are deduced from the kinematic CHAMP positions by means of Gregory-Newton interpolation and numerical differentiation. After reducing tidal and non-conservative disturbing effects, these accelerations are balanced by means of Newton’s Law of Motion with the gradient of a spherical/ellipsoidal harmonic series expansion of the gravitational geopotential. In order to solve the large linear system of equations for the spherical/ellipsoidal harmonic coefficients, the iterative method of preconditioned conjugate gradients is implemented. As simulations and real data analysis exhibit, convergence is reached within 7 to 15 iterations by means of preconditioning. In terms of kinematic orbit preprocessing, wavelet-filtering methods have been tested in order to detect and smooth spikes and outliers out, without disturbing the remaining parts of the signal. Different gravity field solutions up to degree and order 90 were computed with and without regularisation. Comparisons to CHAMP-models obtained by means of different algorithms show that the acceleration approach can compete with other methods for gravity field determination.

1 Introduction

Besides the classical algorithm for orbit analysis, the dynamical (variational) approach (Reigber, 1989; Reigber et al., 2005), alternative methods for gravity field determination from CHAMP data based on kinematic orbits have been developed and applied in recent years. Since kinematic orbits can be determined nowadays with an accuracy of less than 5 cm (Svehla and Rothacher, 2003, 2004), high quality results are obtained from these methods known as short-arc-analysis (Ilk et al., 2005), the energy balance method (Gerlach et al., 2003; Földvary et al., 2005) and the acceleration approach (Austen et al., 2002; Reubelt et al., 2003a,b). All above mentioned methods face advantages and disadvantages and only comparisons of results from simulations and real data analysis (Mayer-Gürr et al., 2005) may detect the best method, which is beyond the scope of this contribution. For instance, the large computational effort due to integration of the variational equations and a nonlinear system of equations can be seen as a disadvantage in view of the dynamical approach, whereas the GPS observations can be handled directly as observations. The short-arc-analysis, where the kinematic positions are treated as observations, circumvents the large computational effort by means of a polynomial for the description of the force function, leading to a linear system of equations. But still the integration of the accelerometer data is problematic, since the infamous bias and tilt of the accelerometer measurements will sum up. The same holds for the energy balance approach, even though the concept of energy conservation offers a good opportunity for accelerometer calibration. Unfortunately, the orbit has to be differentiated once and squared afterwards to obtain the satellite’s kinetic energy, which may increase the noise. No integration of the accelerometer data, but double differentiation of the orbit is performed in terms of the acceleration approach. This step is less critical than it seems to be at first glance, if the noise of the kinematic orbit is correlated. In this case, the noise may be reduced by numerical differentiation, which was already demonstrated by Reubelt et al. (2003a,b). In general, the algorithms based on kinematic orbit analysis can be classified as very economic regarding run time, which is due to the underlying linear system of equations, and especially the acceleration approach can be seen as efficient since no integration of the accelerometer data is necessary and differentiation of the orbits is a fast process. Motivated by the explained advantages and earlier simulations (Reubelt et al., 2003a), real-data analysis was carried out with the acceleration approach. The main outline of the approach is summed up as follows:

Input data are kinematic orbit datasets, which are commonly given in the Conventional Terrestrial System (CTS). To avoid
frame accelerations, these satellite positions have to be transformed into the Conventional Inertial System (CIS) by means of the transformations provided by the International Earth Rotation Service (IERS) (McCarthy, 1996). Applying a 9-point Gregory-Newton interpolation scheme and differentiating the interpolation polynomial twice with respect to time, a time series of interpolated accelerations is extracted from the equidistantly tracked input time series of CHAMP satellite positions. To obtain the pure gravitational accelerations of the Earth, they have to be reduced from gravitational and non-gravitational disturbing effects. Gravitational disturbing accelerations like direct tides, solid Earth tides, ocean tides and atmospheric tides have to be modelled, whereas all non-gravitational disturbing accelerations caused by atmospheric drag, solar radiation pressure or the Earth’s albedo are measured by the CHAMP STAR accelerometer. Finally, the “observed” acceleration vectors are balanced by means of Newton’s Law of Motion with the gradient of the gravitational potential expression in a spherical or ellipsoidal harmonics series expansion. The resulting system of equations is linear with respect to the unknown coefficients of the gravitational potential representation. Detailed formulae describing the acceleration approach can be found in earlier publications (Reubelt et al., 2003a,b).

2 Data Preprocessing

All results of gravity field analysis, which we are going to present here, are based on a two years kinematic CHAMP orbit dataset, which was computed and provided by IAPG/FESG Munich (Svehla and Rothacher, 2004) and covers the observation period from March 2002 to March 2004. Due to the fact that these kinematic orbits are generated purely by evaluation of the GPS phase and pseudo-range observations without application of any force function (dynamic model), they may contain data gaps and outliers (spikes). The latter are mainly caused by changing GPS satellite constellations and often occur close to data gaps. As simulations with synthetic outliers show, the presented acceleration approach is sensitive to the contamination of orbit data with local signal disturbances. Preprocessing of the real kinematic orbit data by means of outlier detection and elimination therefore seems to be advisable and is applicable mainly at two different steps within the whole approach. In order to remove single outliers or small spikes within the original kinematic orbit data set, data preprocessing should be accomplished at the level of the input signal differences. As kinematic orbit disturbances lead to propagated errors in interpolated accelerations, a second option would be to apply outlier removal strategies after the derivation of the interpolated accelerations.

In order to obtain signal representations, which are appropriate for the purpose of filtering methods, difference signals have to be computed in both cases. Subtracting comparatively smooth reduced-dynamic orbits from the kinematic input data set yields a resulting signal-difference, which clearly reveals the outliers within the kinematic orbits.

If interpolated accelerations are considered, it is sufficient to subtract computed model accelerations from existing geopotential models up to degree 2.

For the first computations a very simple method for outlier detection and removal was applied. Here, coordinate differences of consecutive positions (baselines) of both, kinematic and reduced dynamic reference orbit were calculated. A comparison of kinematic and reduced dynamic baselines setting a cut-off limit reveals exceptional outliers, which were eliminated. At the level of accelerations, a second step of outlier-detection was applied. The difference between the “observed” accelerations and accelerations computed from the existing gravity field model EGM96 (Lemoine et al., 1998) up to degree 90 indicates further exceeding outliers. A more elegant and mathematically well-defined method to remove small, temporary occurring outliers from the input data set are wavelet filter techniques, which are based on fast discrete wavelet transformation. Due to their time localizing ability, these are very appropriate for detecting and removing local signal occurrences without effecting the remaining parts of the signal.

By means of the fast discrete wavelet transformation the input signal is developed into a consecutive series expansion of approximation signals and detail signals of increasing scales. Fast wavelet transformation is applicable for orthogonal wavelets with compact support (finite number of corresponding filter coefficients). Daubechies wavelets of order 1 (Haar wavelet) and 2 were applied. All local spikes and outliers within the signal are solely mapped to the coefficients on the smallest scales. Considering multiples of the mean signal energy on these small scales, scale-dependent thresholds are computed. Applying a simple hard-thresholding technique, all wavelet coefficients above these thresholds at each scale are set to zero. In the last step, fast inverse wavelet transformation yields again the original input difference signal, which is now cleaned from the detected outliers. Afterwards the reduced-dynamic orbit, which was subtracted afore, is restored again.

All considered reduced-dynamic orbits result from an orbit adjustment procedure, where pseudo-stochastic pulses, air drag and solar-radiation pressure parameters and initial state vectors are adjusted while the coefficients of a chosen gravitational potential model are considered as a priori information and are not parametrized within the orbit adjustment. This means that the reduced-dynamic orbit contains a predetermined geopotential model field. Fortunately, IAPG/FESG Munich (Svehla and Rothacher, 2004) provided two different two years reduced-dynamic orbit sets, which are based on EIGEN-GRACE02S and EGM96 respectively. In order to guarantee, that the result of the filtering process and more important the result of the following gravitational field analysis does not depend on the parameterisation of the reduced-dynamic orbit, all computations were accomplished using two different kinds of reduced-dynamic orbits.
3 Iterative solution of linear system of equations

For the determination of the gravity field parameters, a system of equations consisting of 6 millions of observations and 8278 unknowns for the maximum degree \( L = 90 \) has to be solved. This may lead to two basic problems, namely the storage of the large design and normal matrices and the time-consuming computation of the normal matrix. The algorithm can be shifted to a super-computer, or an iterative solution can be aimed concerning these problems. Iterative methods are able to deal with restricted memory, since the normal matrix must not be built up and the design matrix must not be stored. If iterative solvers in terms of preconditioned conjugate gradients (PCG) are implemented, the computations can be performed on a standard PC. This method was already described by Hestenes and Stiefel (1952) and Ditmar and Klees (2002) adopted it already successfully for the inversion of simulated GOCE-data. The method of preconditioned conjugate gradients led to a fast and stable convergence in all computations and is able to deal with restricted computer storage. To guarantee convergence within a few iterations, the preconditioner \( N_{\text{pre}} \) should fulfill two conditions, namely: (i) the matrix product \( N_{\text{pre}}^{-1} \cdot N \) is close to the identity matrix \( I \) and (ii) the computation and inversion of the preconditioner should be many times faster than the computation and inversion of the real normal matrix. According to Colombo (1984), the normal matrix becomes blockdiagonal, consisting of one submatrix per order \( m \) (even finer structures can be found), if the orbit fulfills some ideal conditions, for instance it is circular and repeat, it has constant inclination and no data gaps. For a real orbit, these conditions are not fulfilled, so the non-blockdiagonal elements contain small entries instead of zeros. But still the blockdiagonal elements of the normal matrix provide a good preconditioner.

The computation and inversion of the small submatrices of the blockdiagonal preconditioner (Figure 1 (a), see also Ditmar and Klees (2002)) with maximum dimension of \( (L+1) \times (L+1) \) can be done quite fast (condition (ii)), and the storage of these submatrices requires few memory. As Figure 1 (b) exhibits, the matrix product \( N_{\text{pre}}^{-1} \cdot N \) deviates only slightly from the identity matrix \( I \) (condition (i)), so a fast convergence is achieved. As simulations with a one year orbit show, convergence is achieved within 8 - 15 iterations, depending on the orbit height and the resolution of the gravity field (the lower the orbit, the more it deviates from this ideal assumptions due to increasing effects of the higher degree harmonics). For the inversion of a real two years kinematic orbit data set 13 iterations were needed, which is more than for a simulated data set of about the same orbit height. This is not astonishing, since the real orbit deviates more from the "ideal" conditions due to decreasing orbit, the presence of non-conservative forces, orbit manoeuvres and data gaps.

4 Results

Based on the two years kinematic CHAMP orbit, different geopotential models were computed complete up to degree and order 90. The first model GIS_CH01p, was obtained without any regularisation, and only tidal effects were reduced from the satellite’s accelerations. The second model GIS-CH01k results by applying regularisation according to Kaula’s rule and in the third model GIS-CH01p_acc also non-conservative accelerations measured by the STAR- accelerometer are reduced, considering the calibration parameters delivered from GFZ Potsdam. For data preprocessing the simple procedure was chosen. For the evaluation of the determined models, comparisons (Figures 2 - 4) were drawn to GRACE gravity field model EIGEN-GRACE02S (Reigber et al., 2004), which is regarded to be of superior accuracy for degrees up to 100. In Figure 2 (a), differences to EIGEN-GRACE02S are illustrated in terms of degree RMS. Evidently, the gravity field can be determined with significant accuracy up to degree 80 (GIS-CH01p). Regularisation by means of Kaula’s rule improves the resolution up to degree 90 (GIS-CH01k). Astonishingly, the reduction of the accelerometer measurements (GIS_CH01p_acc) is worsening the result instead of improving. This might be due to the fact that the provided accelerometer calibration parameters are not sufficient to remove the bias and tilt of these measurements correctly. Figure 2 (b) additionally illustrates the accumulation of these differences in terms of geoid heights. Comparisons to the latest GFZ CHAMP-model EIGEN-CHAMP03S (Reigber et al., 2005) show, that the proposed method leads to a similar accuracy. Whereas EIGEN-CHAMP03S seems to be closer to EIGEN-GRACE02S for degrees below 45, GIS-CH01k seems to be closer to EIGEN-GRACE02S between degree 60 and 75. Here one has to keep in mind, that EIGEN-CHAMP03S refers to a different observation period of CHAMP data from October 2000 to June 2003. Although almost three years of observations are processed, the older data is less sensitive to higher degrees due to a higher orbit. The triangle-plots of Figure 4 might exhibit the different characteristics of the two methods: While the classical approach applied for EIGEN-CHAMP03S is sensitive to the sectorial and near-sectorial coefficients of higher degree, the acceleration method seems to be more sensitive to zonal and near-zonal coefficients between degree 50 and 75.

The remarkable worse accuracy for degrees 2 - 4 in Figure 2 was confirmed by other groups working with the same kinematic orbits, which leads to the assumption that this is an effect due to the data and not a problem related to the proposed algorithm. The map in Figure 3 displays the geoid height differences between GIS-CH01p and EIGEN-GRACE02S (both models truncated at degree/order 70/70). The differences are more or less equally distributed over the Earth which means that there is no polar data gap visible and no remarkable effects in regions of rough gravity field (e.g. Himalaya). The unweighted RMS over all gridded geoid height differences amounts to 21.6 cm with a maxi-
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Fig. 1. (a) structure of the preconditioner $N_{\text{pre}}$ (b) differences between product matrix $N_{\text{pre}}^{-1} \cdot N$ and the identity matrix for a simulated 86 days CHAMP orbit based on EGM96 up to degree/order 50/50, orbit height $h = 470$ km, sampling interval $\Delta t = 30$ s

Fig. 2. (a) degree RMS difference between the CHAMP-models GIS-CH01p, GIS-CH01k, GIS-CH01p_acc and the GRACE-model EIGEN-GRACE02S (b) accumulated differences to EIGEN-GRACE02S in terms of geoid heights, comparison to EIGEN-CHAMP models

The geoid height differences between EIGEN-CHAMP03S and EIGEN-GRACE02S (not plotted here), also truncated at degree/order 70/70, reach a mean value of 23.4 cm with a maximum of 1.58 m.

5 Conclusions

The analysis of the two years kinematic CHAMP orbit exhibits that the proposed method is able to determine the Earth’s geopotential up to degree/order 80/80 with an accu-
Fig. 3. Geoid height differences (in meter) between GIS-CH01p and EIGEN-GRACE02S (both truncated at degree and order 70): RMS=0.216 m, maximum=1.022 m.

Fig. 4. Relative coefficient differences of the CHAMP-models GIS-CH01k and EIGEN-CHAMP03S compared to EIGEN-GRACE02S.

Accuracy similar to the results of the conventional method, if the kinematic orbit is of high quality. Comparisons with results obtained by alternative approaches based on kinematic orbits (not included here) also show a similar behaviour, proving the high accuracy of the kinematic orbits and the good performance of these algorithms. A combination of these methods might be favourable, since different groups of coefficients are determined more accurate by one approach compared to another. The second order numerical differentiation doesn’t seem to be the weak point of the acceleration approach as was assumed beforehand. This is due to the correlation of the kinematic orbit data. The algorithm can be classified as fast, since the system of equations is linear and numerical differentiation is applied to the data instead of integration. Avoiding integration has the positive side-effect, that no low-frequency-noise is increased. The analysis can be carried out on a standard PC by means of the iterative preconditioned conjugate gradients method, which guarantees fast convergence and is able to cope with restricted memory capacity. All in all, the acceleration approach can be judged as a suited method for gravity field analysis. The fact that already different kinds of acceleration approaches (Fengler et al., 2003; Ditmar et al., 2004) are implemented by other groups...
confirms the high performance of pseudo-observed satellite accelerations. Still work is necessary at certain levels of the algorithm. So far the reduction of the accelerometer-measurements did not improve the results. As the delivered calibration parameters don’t seem to be good enough to remove bias and tilt, the estimation of further calibration parameters should be included in future computations. The algorithm has shown to be sensitive to spikes and jumps in the data. Thus, data preprocessing plays an important role.

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