On the validation of CHAMP- and GRACE-type EGMs and the construction of a combined model

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Abstract

A number of new satellite-only Earth Gravity Models (EGMs) become progressively available based on the CHAMP and GRACE satellite mission data. These models promise higher (compared to older EGMs) accuracy in the determination of the low and medium harmonics of the Earth’s gravity field. In the present study, the latest EGMs generated from CHAMP and GRACE data (namely EIGEN2, EIGEN3p, GGM01C, GGM01S and GRACE01S) have been studied with respect to their accuracy and performance when used in gravity field approximation. A spectral analysis of the new models has been carried out, employing their degree and error-degree variances. In this way, their performance against each other and with respect to EGM96 was assessed, and the parts of the gravity field spectrum that each model describes more accurately have been identified. The results of this analysis led to the development of a combined geopotential model, complete to degree and order 360, whose coefficients were those of CHAMP until degree 5, GRACE until degree 116, and EGM96 for the rest of the spectrum. Finally, a validation of all models (the combined included) has been performed by comparing their estimates against (a) GPS/Levelling data in land areas, (b) TOPEX/Poseidon sea surface heights over sea, and (c) marine and land gravity anomalies. All tests have taken place over Greece and the eastern part of the Mediterranean Sea. From the results obtained it was concluded that the combined EGM developed provides more accurate results (compared to EGM96), in terms of the differences with the control datasets, at the level of 1-2 cm geoid and 1-2 mGal for gravity (1σ).

Furthermore, the absolute geoid accuracy that the combined EGM offers is 12.9 cm (1σ) for n=120, 25 cm for n=200 and 33 cm for n=360, compared to 29 cm, 36 cm and 42 cm for EGM96, respectively.

1 Introduction

The utilization of global Earth Gravity Models (EGMs) in gravity field and geoid determination was and still is a common practice in geodetic studies during the past two decades. EGMs are mostly used to remove the long-wavelength part of the gravity field spectrum when employing the well-known remove-compute-restore method to determine geoidal undulations from, e.g., gravity anomalies and/or altimetric sea surface heights. The internal accuracy of an EGM propagates to the finally estimated geoid heights and thus influences the accuracy of the so-determined geoid model (Tscherning, 2001a). Until recently, the main error source in geoid heights determined by the aforementioned method was induced by the EGM used, since the accuracy of the latter reached the ±50-60 cm level (1σ – standard deviation) for the best available high-resolution model, i.e., EGM96 (Lemoine et al., 1998). The launch of CHAMP and GRACE satellites in July 2000 and March 2002 respectively, signaled a new era in studies related to the estimation of satellite only and combined EGMs,
since they promised enhanced accuracy in the determination of the very-long to long wavelengths of the gravity spectrum. At the present time, four years after the launch of CHAMP, a number of new, satellite-only EGMs have become available based solely on data from CHAMP and GRACE. Most models utilize only a small portion of the data to become available by both satellites in their life span and show already improved accuracies in the low-degree harmonics (see, e.g., Tscherning et al., 2001b).

In the present study, a number of those new-generation EGMs are employed to assess the accuracy improvement that they offer in geoid determination. The models derived from CHAMP data are the GeoForschungsZentrum (GFZ) EIGEN2 (Reigber et al., 2003) and EIGEN3p (Reigber et al., 2005) EGMs, both complete to degree and order 120. The former is determined from about 6 months of CHAMP data compared to three years for the latter. Other CHAMP models are: a) UCPH2003 (Tscherning et al., 2003), derived from one month of the satellite’s data and complete to degree and order 90, b) TUM2Sp (Földvary et al., 2005), derived from one year of the satellite’s data and complete to degree and order 70, and finally c) ITG_Champ01E (Ilk et al., 2005), derived from one year of the satellite’s data and complete to degree and order 75. In the GRACE-based EGM front end three models were used, namely the Center for Space Research (CSR) GGM01S (Tapley et al., 2003) and GGM01C (Tapley et al., 2004) both based on 111 days data of the satellite and complete to degree and order 120 and 200, respectively. GGM01S is a satellite only solution, while GGM01C is its combined counterpart. Finally, GRACE01S is a satellite only model based on 49 days of GRACE data and complete to degree and order 140.

Apart from these models, a number of old EGMs, i.e., EGMs compiled during the previous years using satellite tracking methods, altimetry and surface gravity data, were employed as well, to assess the improvement that the latest EGMs offer. The former were EGM96 and EGM96S complete to degree and order 360 and 70, respectively (Lemoine et al., 1996).

2 Spectral analysis of geopotential models

The processing methodology was based on the spectral analysis of the available EGMs from CHAMP and GRACE to determine those that describe more accurately the various frequencies of the gravity field spectrum. Then, a so-called combined model is determined by employing for each degree the coefficients of that EGM which proved to be the most accurate from the previous analysis.

All models come as a series of spherical harmonic coefficients, of various degrees and orders, together with the errors associated for each coefficient. Therefore, for all models the harmonic coefficients $c_{nm}$, $s_{nm}$ and their accuracies $\sigma_{c_{nm}}$, $\sigma_{s_{nm}}$ are provided. Based on these, the signal and error degree variances for each model, either per degree or cumulatively, can be computed. The models available represent spherical harmonic expansions of the Earth’s disturbing potential, therefore the so-determined signal and error degree variances refer to that. Nevertheless, they can be easily converted to represent various quantities related to the Earth’s gravity field, such as gravity anomalies, geoid heights, etc. Since the main interest in using an EGM is in geoid or gravity field determination, it has been decided to validate the available models, with respect to (w.r.t.) the accuracy they provide in geoid heights and gravity anomalies. The signal degree variances represent the amount of the signal contained in each degree or up to a specific degree (if computed cumulatively), while the error degree variances represent the error of the model up to a specific degree.

Since various geopotential models were available and needed to be compared, it was necessary to scale their harmonic coefficients, so that they will all refer to the surface of a
sphere of radius $R$. In that way, the computed signal and error degree variances are comparable. The scaled signal and error degree variances for the various quantities related to the gravity field can be computed as follows (Pavlis, 1998):

a) For the disturbing potential

$$
\sigma^2_n = \left( \frac{GM}{a} \right)^2 \left( \frac{a^2}{R^2} \right)^{n+1} \sum_{m=0}^{n} \left( C_{nm}^2 + S_{nm}^2 \right),
$$

(1)

$$
\epsilon^2_n = \left( \frac{GM}{a} \right)^2 \left( \frac{a^2}{R^2} \right)^{n+1} \sum_{m=0}^{n} \left( \epsilon^2_{nm} + \epsilon^2_{nm} \right),
$$

(2)

b) for gravity anomalies

$$
\sigma^2_n = \left( \frac{GM}{a} \right)^2 \left( n-1 \right)^2 \left( \frac{a^2}{R^2} \right)^{n+1} \sum_{m=0}^{n} \left( C_{nm}^2 + S_{nm}^2 \right),
$$

(3)

$$
\epsilon^2_n = \left( \frac{GM}{a} \right)^2 \left( n-1 \right)^2 \left( \frac{a^2}{R^2} \right)^{n+1} \sum_{m=0}^{n} \left( \epsilon^2_{nm} + \epsilon^2_{nm} \right),
$$

(4)

c) for geoid heights

$$
\sigma^2_n = \left( \frac{GM}{\gamma a} \right)^2 \left( \frac{a^2}{R^2} \right)^n \sum_{m=0}^{n} \left( C_{nm}^2 + S_{nm}^2 \right),
$$

(5)

$$
\epsilon^2_n = \left( \frac{GM}{\gamma a} \right)^2 \left( \frac{a^2}{R^2} \right)^n \sum_{m=0}^{n} \left( \epsilon^2_{nm} + \epsilon^2_{nm} \right),
$$

(6)

Using Eqs. 3-6 the signal and error degree variances for the various EGMs from CHAMP and GRACE have been computed (see Figs. 1 and 2). From Fig. 1 it is concluded that GGM01C has the same power as EGM96 up to its maximum degree of expansion ($n=200$), while it retains full power up to about $n=112$. Its error is smaller than that of EGM96 up to $n=120$ and the accuracy improvement that it offers, compared to the latter, is about 20 times better (see Fig. 2). GGM01C offers a ±1 cm accuracy up to $n=62$ while it reaches the ±10 cm level at $n=112$. The GGM01S model retains full power up to $n=95$, while its accuracy improvement, compared to EGM96, is the same as that of GGM01C up to $n=70$. The corresponding degrees that GGM01S reaches the ±1 and ±10 cm level of accuracy are $n=52$ and $n=101$. Finally, GRACE01S retains full signal power up to $n=95$ and offers a ±1 cm accuracy up to $n=52$ while it reaches the ±10 cm level at $n=90$. 

3
Figure 1. Geoid signal and error degree variances from the various models.

As far as the CHAMP-derived EGMs are concerned, EIGEN2 retains full power up to \( n=30 \) and the improvement in the geoid accuracy that it offers, compared to EGM96, is inferior to the GRACE models. From Fig. 2 it can be seen that EIGEN2 is about two times more accurate than EGM96 and gives accuracies of \( \pm 1 \) and \( \pm 10 \) cm up to \( n=20 \) and \( n=39 \), respectively. The EIGEN3p EGM, which is a preliminary model, is about two times more accurate than EIGEN2, which is due to the use of a longer time-series of CHAMP data in its development. It retains full power to \( n=52 \) while it reaches the 1 and 10 cm accuracies to \( n=27 \) and \( n=56 \), respectively. Finally, the UCPH2003 EGM retains full power to degree \( n=47 \) and its accuracy is between the two GFZ models (see Fig. 2). The 1 cm accuracy is achieved up to \( n=25 \) and the 10 cm one up to \( n=51 \). From the analysis given so far, the best model that is developed from satellite data alone is GGM01S, while the best combined solution is GGM01C. These models give a \( \pm 1 \) cm accuracy up to wavelengths of 380 and 319 km, respectively (half wavelength), while the \( \pm 10 \) cm accuracy is retained up to wavelengths as short as 196 and 176 km, respectively. For EGM96, the corresponding wavelengths are at the 2830 and 550 km respectively, while for EIGEN3p they reach the 733 and 354 km. From these results it is clear that the accuracy improvement that the new satellite models offer is significant and taking into account that the development of the CSR GRACE models was based on only 111 days of satellite data, we can expect far better results when new data sets become available.
After that step, a further analysis has been performed by dividing the geoid spectrum in wave bands of 20 degrees from 0 up to 360 (see Table 1). In each waveband the accuracy that the different models offer has been assessed and in cases where their performance was ambiguous, the analysis has been contacted by degree. From Table 1 we can conclude that EIGEN 2 provides the most accurate results for degrees 1-5, while the superiority of the GRACE-based models for degrees 6-116 is obvious. This fact signals the scope that each satellite was built for, i.e., that CHAMP intends to accurately map the gravity field at the very low harmonic degrees, while GRACE the long to medium part of the spectrum. From the GRACE models, the one that provides the best accuracy for the degrees 6-116 is GGM01C, while above that degree, the EGM96 model gives the best results.

Table 1. Cumulative geoid error from the different EGMs in wavebands of 20 degrees.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>Harmonic Degrees – Wavebands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-20</td>
</tr>
<tr>
<td>EIGEN2</td>
<td>1.04</td>
</tr>
<tr>
<td>EIGEN3p</td>
<td>0.63</td>
</tr>
<tr>
<td>GGM01C</td>
<td>0.16</td>
</tr>
<tr>
<td>GGM01S</td>
<td>0.33</td>
</tr>
<tr>
<td>GRACE0</td>
<td>0.36</td>
</tr>
<tr>
<td>EGM96</td>
<td>4.91</td>
</tr>
</tbody>
</table>
3 Determination of the combined model

After the spectral analysis of the available EGMs, a so-called combined geoid model has been determined by using for each degree the harmonic coefficients of the CHAMP or GRACE-type EGM that provided the best accuracy (for the specific degree). Therefore, the combined model was determined as

\[ N^{\text{GM}} = N_{\text{CHAMP}} + N_{\text{GRACE}} + N_{\text{EGM96}}, \]  

where \( N^{\text{GM}} \) is the total contribution of the EGMs, i.e., the combined model, and \( N_i \) is the contribution of the CHAMP, GRACE or EGM96 geopotential models to specific degrees, correspondingly. It is important to mention that the contribution of EGM96 is needed so as to develop a highly-expanded EGM, since an EGM complete to degree and order, e.g., 200, is of little use for geoid determination because it resolves wavelengths only up to 198 km. The \( N_i \) for each model will successfully provide geoid heights according to the following equations:

\[ N_{\text{CHAMP}} = \frac{kM}{r^\gamma} \sum_{n=2}^{5} \left( \frac{a}{r} \right)^n \sum_{m=0}^{n} \left( \bar{C}_n^\text{CHAMP} \cos m\lambda + \bar{S}_n^\text{CHAMP} \sin m\lambda \right) \bar{P}_m (\sin \phi), \]  

\[ N_{\text{GRACE}} = \frac{kM}{r^\gamma} \sum_{n=6}^{116} \left( \frac{a}{r} \right)^n \sum_{m=0}^{n} \left( \bar{C}_n^\text{GRACE} \cos m\lambda + \bar{S}_n^\text{GRACE} \sin m\lambda \right) \bar{P}_m (\sin \phi), \]  

\[ N_{\text{EGM96}} = \frac{kM}{r^\gamma} \sum_{n=117}^{360} \left( \frac{a}{r} \right)^n \sum_{m=0}^{n} \left( \bar{C}_n^\text{EGM96} \cos m\lambda + \bar{S}_n^\text{EGM96} \sin m\lambda \right) \bar{P}_m (\sin \phi), \]

Based on this methodology, a combined EGM has been determined and its signal and error degree variances have been estimated. From the analysis performed it was found that the total geoid height error of the new model was at the order of 33 cm (compared to 50 cm for EGM96), while the ±1 cm and ±10 cm accuracies were achieved up to degrees \( n=62 \) and \( n=112 \) respectively (compared to \( n=7 \) and \( n=37 \) for EGM96). It is obvious from these results that the newly combined EGM, which is based on CHAMP and GRACE data, presents a much more accurate picture of the Earth’s gravity field. If the new EGM was truncated only to degree and order 120, then its accuracy would reach the 12 cm only, but a high-degree model is necessary to be used as a reference field for gravity data and altimetric observations for use in geoid determination.

The so-determined EGM provides a new “combined” geoid model (see Fig. 3) as well as a gravity field for the area under study, with their statistics shown in Table 2. In Table 2 the contributions of a) EIGEN2 to degrees 2-5, b) GGM01C to degrees 6-116 and c) EGM96 to degrees 117-360 are also given. From that Table it can be concluded that the main part of the geoid signal is contained in the very-long and long wavelengths (44% of the total signal is provided up to \( n=6 \)) while the short wavelengths contribute only up to 8.7% of the total signal. On the contrary, for gravity anomalies, only 2% of the total signal is contained up to degree \( n=6 \) while the main part is provided by the medium wavelengths.
Figure 3. The new “combined” geoid model in the area under study.
Table 2. Statistics of geoid heights and gravity anomalies form the new combined EGM and the contributions of the geopotential models used.

<table>
<thead>
<tr>
<th>Model</th>
<th>max</th>
<th>min</th>
<th>mean</th>
<th>rms</th>
<th>std</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMB (m)</td>
<td>47.859</td>
<td>0.819</td>
<td>31.467</td>
<td>33.719</td>
<td>±12.117</td>
</tr>
<tr>
<td>NEIGEN (m)</td>
<td>45.454</td>
<td>21.549</td>
<td>34.453</td>
<td>34.847</td>
<td>±5.299</td>
</tr>
<tr>
<td>GGM01C (m)</td>
<td>13.067</td>
<td>-24.611</td>
<td>-2.975</td>
<td>10.673</td>
<td>±10.250</td>
</tr>
<tr>
<td>EGM96 (m)</td>
<td>4.676</td>
<td>-4.353</td>
<td>-0.011</td>
<td>1.043</td>
<td>±1.043</td>
</tr>
<tr>
<td>Dg COMB (mGal)</td>
<td>116.147</td>
<td>-198.85</td>
<td>3.471</td>
<td>62.089</td>
<td>±61.992</td>
</tr>
<tr>
<td>Dg EIGEN (mGal)</td>
<td>13.516</td>
<td>7.909</td>
<td>10.650</td>
<td>10.718</td>
<td>±1.201</td>
</tr>
<tr>
<td>GGM01C (mGal)</td>
<td>112.503</td>
<td>-162.372</td>
<td>-6.788</td>
<td>56.967</td>
<td>±56.561</td>
</tr>
<tr>
<td>EGM96 (mGal)</td>
<td>135.937</td>
<td>-136.327</td>
<td>-0.391</td>
<td>27.105</td>
<td>±27.102</td>
</tr>
</tbody>
</table>

4 Validation of the combined EGM

To assess the accuracy of the CHAMP and GRACE derived EGMs as well as that of the combined model, comparisons with 130 GPS/Leveling geoid heights have been performed. Fig. 4 shows the distribution of the GPS/Leveling benchmarks (BM), located in Northern Greece in the wider area of Thessaloniki. For the minimization of the differences between the EGM and GPS/Leveling geoid heights different parametric models have been used namely a) a simple mean removal model, b) a 1st order polynomial model, c) a 2nd order polynomial model, d) a 3rd order polynomial model, and e) the classic four-parameter transformation model corresponding to a datum transformation. First, the performance of the parametric models has been assessed and the one that provides the smallest residual in terms of the standard deviation of the differences ($1\sigma$) has been selected. Using that criterion, the 3rd order polynomial model (see Eq. 11) has been selected as the most appropriate one.

\[
N^\text{GPS} - N^\text{GM} = x_1 + x_2 \left( \phi_1 - \bar{\phi} \right) + x_3 \left( \lambda_1 - \bar{\lambda} \right) + x_4 \left( \phi_1 - \bar{\phi} \right)^2 + x_5 \left( \lambda_1 - \bar{\lambda} \right)^2 +
+ x_6 \left( \phi_1 - \bar{\phi} \right) \left( \lambda_1 - \bar{\lambda} \right) + x_7 \left( \phi_1 - \bar{\phi} \right)^3 + x_8 \left( \lambda_1 - \bar{\lambda} \right)^3 +
+ x_9 \left( \phi_1 - \bar{\phi} \right)^2 \left( \lambda_1 - \bar{\lambda} \right) + x_9 \left( \phi_1 - \bar{\phi} \right) \left( \lambda_1 - \bar{\lambda} \right)^2 + v,
\]  

Then the differences between the GPS/Leveling geoid heights and those estimated by the EGMs have been compared, with the results being summarized in Table 3. Before the fit of the parametric model, UCHP2003 and EIGEN3p provide the smallest differences ($\sigma$ at the ±33 and ±40 cm respectively compared to ±41 cm for the combined model). This can be misleading w.r.t. the performance of the models if one does not consider the mean value of the differences as well, which for the aforementioned models is about 2 m and 20 cm larger than that of the combined model. Therefore it can be concluded that the new EGM provides more accurate results w.r.t. UCHP2003 and EIGEN3p. As far as GGM01C is concerned, its differences with the GPS data are at the ±48 cm which is 7 cm worst than that of the new
combined model. After the fit it can be seen from Table 3 that the combined EGM provides the best results, i.e., smaller $\sigma$ and range by about 2% and 3% respectively compared to the other models.

Table 3. Geoid height differences between $N^{GPS/Lev}$ and $N^{GM}$ before and after the fit of a 3rd order polynomial model. [m]

<table>
<thead>
<tr>
<th></th>
<th>max</th>
<th>min</th>
<th>mean</th>
<th>rms</th>
<th>std</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{GPS} - N^{COMB}$</td>
<td>-0.347</td>
<td>-2.469</td>
<td>-1.072</td>
<td>1.150</td>
<td>±0.415</td>
</tr>
<tr>
<td></td>
<td>0.482</td>
<td>-1.186</td>
<td>0.000</td>
<td>0.215</td>
<td>±0.215</td>
</tr>
<tr>
<td>$N^{GPS} - N^{EGM96}$</td>
<td>-0.821</td>
<td>-2.662</td>
<td>-1.341</td>
<td>1.410</td>
<td>±0.435</td>
</tr>
<tr>
<td></td>
<td>0.483</td>
<td>-1.185</td>
<td>0.000</td>
<td>0.221</td>
<td>±0.221</td>
</tr>
<tr>
<td>$N^{GPS} - N^{EIGEN2}$</td>
<td>0.630</td>
<td>-2.821</td>
<td>-1.064</td>
<td>1.313</td>
<td>±0.770</td>
</tr>
<tr>
<td></td>
<td>0.491</td>
<td>-1.234</td>
<td>0.000</td>
<td>0.221</td>
<td>±0.221</td>
</tr>
<tr>
<td>$N^{GPS} - N^{EIGEN3p}$</td>
<td>-0.427</td>
<td>-2.503</td>
<td>-1.202</td>
<td>1.266</td>
<td>±0.397</td>
</tr>
<tr>
<td></td>
<td>0.499</td>
<td>-1.244</td>
<td>0.000</td>
<td>0.222</td>
<td>±0.222</td>
</tr>
<tr>
<td>$N^{GPS} - N^{UCPH2003}$</td>
<td>-2.213</td>
<td>-4.513</td>
<td>-3.164</td>
<td>3.181</td>
<td>±0.329</td>
</tr>
<tr>
<td></td>
<td>-1.245</td>
<td>0.501</td>
<td>0.000</td>
<td>0.223</td>
<td>±0.223</td>
</tr>
<tr>
<td>$N^{GPS} - N^{GGM01C}$</td>
<td>-0.046</td>
<td>-2.510</td>
<td>-0.931</td>
<td>1.048</td>
<td>±0.482</td>
</tr>
<tr>
<td></td>
<td>0.481</td>
<td>-1.226</td>
<td>0.000</td>
<td>0.219</td>
<td>±0.219</td>
</tr>
<tr>
<td>$N^{GPS} - N^{GGM01S}$</td>
<td>-0.169</td>
<td>-3.810</td>
<td>-1.742</td>
<td>1.893</td>
<td>±0.740</td>
</tr>
<tr>
<td></td>
<td>0.508</td>
<td>-1.254</td>
<td>0.000</td>
<td>0.224</td>
<td>±0.224</td>
</tr>
<tr>
<td>$N^{GPS} - N^{GRACE01S}$</td>
<td>-0.051</td>
<td>-3.225</td>
<td>-1.363</td>
<td>1.484</td>
<td>±0.588</td>
</tr>
<tr>
<td></td>
<td>0.504</td>
<td>-1.250</td>
<td>0.000</td>
<td>0.223</td>
<td>±0.223</td>
</tr>
</tbody>
</table>

Figure 4. Distribution of GPS/Leveling BMs used for the EGM validation.
5 Conclusions

An analysis of the performance of some CHAMP- and GRACE-based EGMs on geoid and gravity field determination has been presented. From the results obtained it was concluded that the CHAMP models provide the most reliable and accurate results for the very-long wavelengths (up to degree n=5), the GRACE models are superior up to degree n=116, while EGM96 remains the dominant geopotential model for the shorter wavelengths.

From the spectral analysis of the geopotential models coming from the new satellite missions, a new combined EGM was determined using for each degree the coefficients of that EGM which was more accurate. The so-determined combined geopotential model outperforms EGM96 and the other models, since it provides the smallest differences when compared with GPS/Leveling geoid heights. Before the fit of a parametric model, it provides smaller differences (1σ) by about ±2 and ±7 cm compared to EGM96 and GGM01C, while the range of the differences was 67 cm smaller than that of EGM96. After the fit, the combined model provides smaller σ and by about 2%, compared to the other models, while the magnitude of the differences ranges by about 3% less.

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References


