Gravity Field Modelling From CHAMP Kinematic Orbits Using the Energy Balance Approach


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Abstract. A gravity field model has been computed based solely on CHAMP GPS orbit tracking and accelerometry, without any prior gravity information entering the computation. The basic characteristics of our approach are (1) the use of purely kinematic orbits, (2) a translation of velocities to gravity potential employing the principle of energy conservation and (3) correction for non-gravitational forces either derived from measured accelerations or from models. The method of gravity field recovery using energy considerations has recently been tested successfully by various groups, and it has been shown, that it is possible to derive a solution only from a purely kinematic CHAMP orbit without any use of prior knowledge. Thereby the satellite velocities are determined separately from the kinematic positions. Now two recent years of CHAMP data have been used to derive the gravity field model TUM-2S. The quality of the field has been tested by comparing the model to other state-of-the-art models as well as to terrestrial data.

1 Introduction

Gravity Field determination by the Energy Balance Approach, as it was applied in this study, consists of four main steps:

1. Precise Kinematic Orbit Determination
2. Derivation of Kinematic Velocities
3. Gravity Modelling by the Energy Integral
4. Least Squares Adjustment of Potential Coefficients.

For the Kinematic POD, two years of GPS measurements (from DoY 70/2002 to DoY 70/2004) of the CHAMP onboard receiver Black Jack have been used, cf. (Švehla and Rothacher, 2003a) and (Švehla and Rothacher, 2003b). In contrast to dynamic (or reduced-dynamic) orbit determination, kinematic POD is based on GPS-measurements only, and no a priori models enter the computation. In a previous study (Gerlach et. al., 2003), it could be shown, that gravity field determination using the energy balance approach with reduced-dynamic orbits, produces gravity field models, which are strongly influenced by the a priori gravity field model, that has been used for POD. The disadvantage of kinematic POD is, that it only delivers positions, and no velocities. As velocities are the core quantity of the energy balance approach, they have to be derived numerically from positions.

The Energy Balance Approach is based on the energy conservation law, which states, that in a conservative force field, the sum of potential and kinetic energy is conserved. As CHAMP is subject to non-conservative forces, reality is more complicated, and the disturbing potential $T$ is derived from the following model:

$$ T = \frac{1}{2} \dot{x}^2 - U - \frac{1}{2} (\omega \times x)^2 - \int_x a_f - \int_x a_t - C . \quad (1) $$

As we choose the disturbing potential $T$ to enter the adjustment as pseudo-observation, the normal potential $U$ has to be subtracted from the kinetic energy. The computation takes place in a rotating Earth-fixed reference frame, so a centrifugal potential $E_{cent} = 1/2 (\omega \times x)^2$ has to be computed and corrected for.

The accelerations $a_f$ caused by external surface forces, like air-drag or solar radiation pressure, are measured by the onboard STAR-accelerometer. They have to be integrated along the orbit and subtracted from the disturbing potential. Also tidal accelerations $a_t$ have to be integrated along the orbit and subtracted, as the satellite is moving in a time-varying gravity field, and thus the integrated potential is path-dependent.
2 Velocity Determination

Kinematic orbit determination results in a time series of 3D-positions. Velocities have to be determined from them by numerical differentiation. They play a central role, from which the kinetic energy is derived: \( E_{\text{kin}} = \frac{1}{2} \dot{x}^2 \). There is a large variety of algorithms available for numerical differentiation, each with different characteristics, and spectral behavior. The aim is to find an algorithm which smoothes the position noise as good as possible, and conserves gravity information, to a maximum extent. Figure 1 shows an ideal filter, a first order, third order, and 22nd order FIR filter. The ideal filter corresponds to a multiplication of the spectrum by \( i\omega \). It is ideal in theory, but problematic with real data, as data gaps and outliers cause large edge-effects. The first order filter is the most simple differentiator: \( \dot{x}_i = (x_{i+1} - x_{i-1})/\Delta t \). We found the third order filter to be a good compromise between high frequency error smoothing, and conservation of the gravity signal. It corresponds to a 7-point Newton-Gregory differentiation:

\[
\dot{x}_i = \frac{-x_{i-3} + 9x_{i-2} - 45x_{i-1} + 45x_{i+1} - 9x_{i+2} + x_{i+3}}{60\Delta t}
\] (2)

Fig. 1. Magnitude Response of Different Differentiators.

3 Gravity Field Modelling

The data used for this study contains 2 years of measurements, with a sampling rate of 30 s, so the number of observations is nearly 2 million. The orbit height starts at about 400 km and drops steadily due to the drag. During two maneuvers in 2002 the satellite was lifted again, and after the second maneuver, the eccentricity was set to nearly zero, so that the orbit height variations during one revolution were reduced. These characteristics are reflected in figure 2.

Figure 3 shows the kinetic energy (in blue) and the normal potential (in red). From both signals, the mean, which is taken as rough estimate of the unknown energy constant) has been subtracted. The latter is strongly dependent on the orbit height. The difference of both signals is shown in figure 4 (in blue) and compared to the centrifugal potential (in red). After subtracting the centrifugal potential, only the unknown disturbing potential and non-conservative signals remain, which is shown in figure 5 (in blue). One can clearly see the loss of energy, due to external forces. The measured accelerations – integrated along the orbit (in red) show a very similar behavior, but differ in detail. The amplitude of the two orbit maneuvers is not measured correctly, and the two curves are not exactly parallel. The difference plotted in figure 6 (in blue) still contains the influence of tides. It shows mainly two periodic signals: a low-frequency signal caused by the sun, and a 14-day periodic signal caused by the moon. The red curve shows the tidal accelerations computed from models and integrated along the orbit.

The remaining signal - after all reductions have been applied - still exhibits large jumps, drifts and changing drifts. The jumps occur mostly after orbit data gaps or accelerometer data gaps, where the energy loss due to external forces cannot be computed continuously. The signal drift is most likely caused by a changing accelerometer bias, and can be estimated in several ways. We have applied the method of fitting piecewise long-wavelength polynomials to the signal, and subtracting them. So we end up with an expected signal for the disturbing potential in orbit height, varying from -800 to 600 m²/s², which enters the adjustment cf. figure 8.

4 Results

The spherical harmonic potential coefficients have been computed by least-squares adjustment. The maximum degree \( L_{\text{max}} \) has been set to 60, as comparisons to other models (e.g. EIGEN-GRACE01) show, that the signal content is no longer significant for higher degrees (cf. figure 9). Figure 10 shows a comparison to the EIGEN-GRACE01 model produced by
GFZ, in geoid heights on a global grid. Both models have been used up to degree 60, and the global root-mean-square (RMS) of the difference is 0.162 m.

For the purpose of external validation, the TUM-2S model and other models have been compared to three sets of GPS-levelling geoid heights from the USA with 5168 points, Canada with 1443 points and Europe (EUREF) with 180 points. To be comparable, all models were used up to degree 60. The GPS-levelling geoid heights were filtered with the GPM98A model (Wenzel, 1998) from degree 61 to 720, so that primarily the low-frequent components are compared. The RMS differences around the mean are shown in table 1.

It can be seen, that the TUM-2S model is a large improvement over our first model TUM-1S. It also fits better to the GPS-levelling data, than the EIGEN-2 model, but not quite as well as EIGEN-3. The best of the tested models is EIGEN-GRACE01, which was derived from 66 days of GRACE measurements.
There are still some improvements feasible, which have to be considered in the future. First, the accelerometer behavior has to be better understood in order to model biases and drifts more accurately. Second, a proper stochastic model has to be introduced, to deal with correlations between GPS-positions, and correlations introduced by the velocity derivation. This has been neglected so far, due to the large computational effort.

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References


5 Conclusions

The results show, that the Energy Balance Approach with kinematic orbits is a valid method for the computation of gravity field solutions. It will also be employed for the GPS part of the ESA mission GOCE, which will be launched in 2006.